

# Spatiotemporal Mixed Modeling of task fMRI

Ben Risk, SAMSI and UNC-CH Biostatistics  
David Matteson and David Ruppert, Cornell University

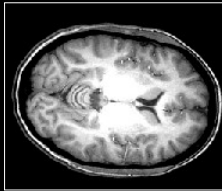
September 25, 2015



# Overview of fMRI

high resolution  
(1 mm)

MRI

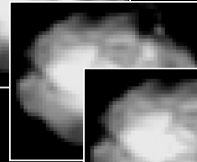


one image (no time)

fMRI



low resolution  
(~3 mm)



many images  
(e.g., every 2 sec for 5 mins)

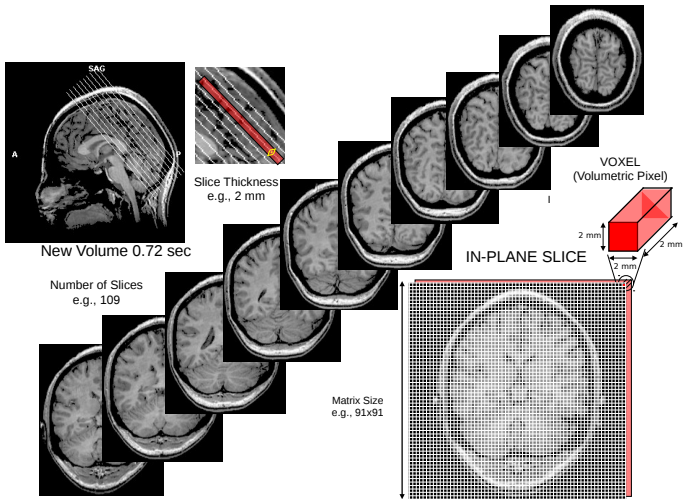
fMRI

Blood Oxygenation Level Dependent (BOLD) signal  
-indirect, “sluggish” measure of neural activity

↑ neural activity → ↑ blood oxygen → ↑ fMRI signal

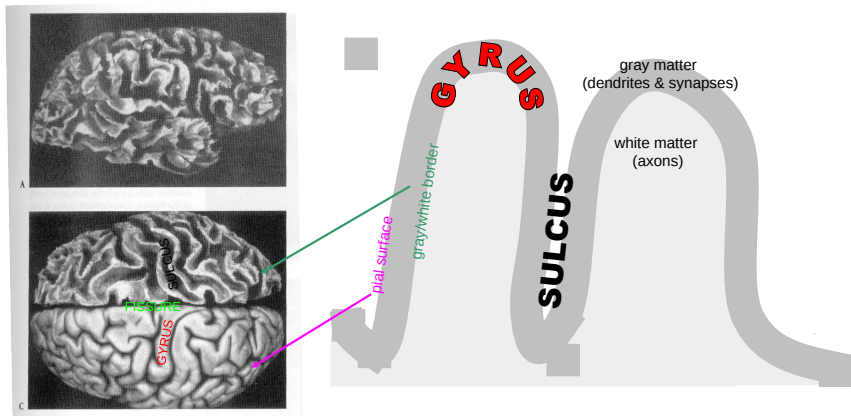
Adapted from: [http://psy.mq.edu.au/vision/~peterw/corella/315/fMRI\\_part1.pdf](http://psy.mq.edu.au/vision/~peterw/corella/315/fMRI_part1.pdf)

# fMRI volumes



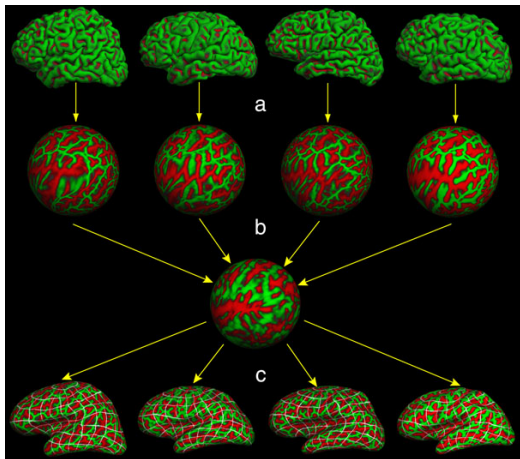
Adapted from: [http://psy.mq.edu.au/vision/~petenw/corella/315/fMRI\\_part1.pdf](http://psy.mq.edu.au/vision/~petenw/corella/315/fMRI_part1.pdf)

# Topology of the brain



Adapted from: [http://psy.mq.edu.au/vision/~peterw/corella/315/fMRI\\_part1.pdf](http://psy.mq.edu.au/vision/~peterw/corella/315/fMRI_part1.pdf)

# Surface registration: vertices



# Challenges of fMRI

- Our example dataset:  
 $N = 20$ , 2 sessions,  $T = 274$ , Voxel dimensions =  $91 \times 109 \times 91$ ;
  - 9.9 billion data points

# Challenges of fMRI

- Our example dataset:  
 $N = 20$ , 2 sessions,  $T = 274$ , Voxel dimensions =  $91 \times 109 \times 91$ ;
  - 9.9 billion data points
- After masking, we have  
 $N = 20$ , 2 sessions,  $T = 274$ , 230k voxels
  - 2.5 billion data points

# Challenges of fMRI

- Our example dataset:  
 $N = 20$ , 2 sessions,  $T = 274$ , Voxel dimensions =  $91 \times 109 \times 91$ ;
  - 9.9 billion data points
- After masking, we have  
 $N = 20$ , 2 sessions,  $T = 274$ , 230k voxels
  - 2.5 billion data points
- If we restrict ourselves to right cerebral cortex,  $N = 20$ , 2 session,  $T = 274$ , 30k vertices
  - 326 million data points

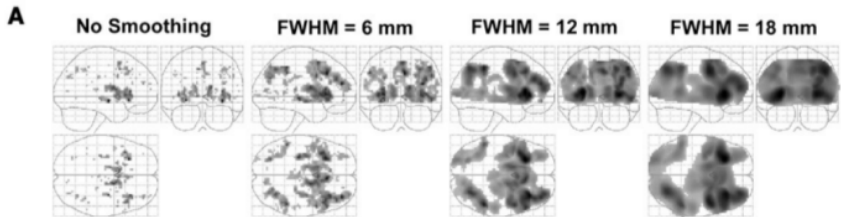
# Motivation

Most popular approach: Massive Univariate Mixed Model (MUMM)

- Separate mixed model estimated for each voxel
- Spatial dependence can be accounted for post-estimation using random field theory
- Data are spatially smoothed with a Gaussian kernel during pre-processing to:
  1. Increase overlap of features between subjects
  2. Increase power and meet assumptions of random field theory
- Conflict: increase power but decrease precision of localization
- So how much smoothing? Not clear.

# Smoothing affects inference

*M. Mikl et al. / Magnetic Resonance Imaging 26 (2008) 490–503*



# Literature Review of Spatial Models

1. [Bowman, 2007] used subject-vertex random effects with exponential covariogram and functionally defined distance.

# Literature Review of Spatial Models

1. [Bowman, 2007] used subject-vertex random effects with exponential covariogram and functionally defined distance.
  - Data application:  $N = 12$ ,  $V = 239$ ,  $T = 4$  (PET data)

# Literature Review of Spatial Models

1. [Bowman, 2007] used subject-vertex random effects with exponential covariogram and functionally defined distance.
  - Data application:  $N = 12$ ,  $V = 239$ ,  $T = 4$  (PET data)
2. [Bernal-Rusiel et al., 2013] extended [Bowman, 2007] to cortical thickness using geodesic distances

# Literature Review of Spatial Models

1. [Bowman, 2007] used subject-vertex random effects with exponential covariogram and functionally defined distance.
  - Data application:  $N = 12$ ,  $V = 239$ ,  $T = 4$  (PET data)
2. [Bernal-Rusiel et al., 2013] extended [Bowman, 2007] to cortical thickness using geodesic distances
  - $N \approx 800$ ,  $V = 149,000$ ,  $T \approx 5$ . Defined novel parcellation with 12 vertices/parcel, parcels independent (12,000 parcels!). FWHM=15mm.

# Literature Review of Spatial Models

1. [Bowman, 2007] used subject-vertex random effects with exponential covariogram and functionally defined distance.
  - Data application:  $N = 12$ ,  $V = 239$ ,  $T = 4$  (PET data)
2. [Bernal-Rusiel et al., 2013] extended [Bowman, 2007] to cortical thickness using geodesic distances
  - $N \approx 800$ ,  $V = 149,000$ ,  $T \approx 5$ . Defined novel parcellation with 12 vertices/parcel, parcels independent (12,000 parcels!). FWHM=15mm.
3. [Derado et al., 2010] assumed constant correlation within Brodmann Area, independent BAs

# Literature Review of Spatial Models

1. [Bowman, 2007] used subject-vertex random effects with exponential covariogram and functionally defined distance.
  - Data application:  $N = 12$ ,  $V = 239$ ,  $T = 4$  (PET data)
2. [Bernal-Rusiel et al., 2013] extended [Bowman, 2007] to cortical thickness using geodesic distances
  - $N \approx 800$ ,  $V = 149,000$ ,  $T \approx 5$ . Defined novel parcellation with 12 vertices/parcel, parcels independent (12,000 parcels!). FWHM=15mm.
3. [Derado et al., 2010] assumed constant correlation within Brodmann Area, independent BAs
  - Use coefficients from single-subject analysis,  $N = 27$ , 2 sessions,  $53 \times 63 \times 46$ ,  $T = \text{hundreds}$

# Literature Review of Spatial Models

1. [Bowman, 2007] used subject-vertex random effects with exponential covariogram and functionally defined distance.
  - Data application:  $N = 12$ ,  $V = 239$ ,  $T = 4$  (PET data)
2. [Bernal-Rusiel et al., 2013] extended [Bowman, 2007] to cortical thickness using geodesic distances
  - $N \approx 800$ ,  $V = 149,000$ ,  $T \approx 5$ . Defined novel parcellation with 12 vertices/parcel, parcels independent (12,000 parcels!). FWHM=15mm.
3. [Derado et al., 2010] assumed constant correlation within Brodmann Area, independent BAs
  - Use coefficients from single-subject analysis,  $N = 27$ , 2 sessions,  $53 \times 63 \times 46$ ,  $T = \text{hundreds}$
4. [Kang et al., 2012] spatial modeling in spectral (time) domain allowing for correlated regions of interest (ROIs)

# Literature Review of Spatial Models

1. [Bowman, 2007] used subject-vertex random effects with exponential covariogram and functionally defined distance.
  - Data application:  $N = 12$ ,  $V = 239$ ,  $T = 4$  (PET data)
2. [Bernal-Rusiel et al., 2013] extended [Bowman, 2007] to cortical thickness using geodesic distances
  - $N \approx 800$ ,  $V = 149,000$ ,  $T \approx 5$ . Defined novel parcellation with 12 vertices/parcel, parcels independent (12,000 parcels!). FWHM=15mm.
3. [Derado et al., 2010] assumed constant correlation within Brodmann Area, independent BAs
  - Use coefficients from single-subject analysis,  $N = 27$ , 2 sessions,  $53 \times 63 \times 46$ ,  $T = \text{hundreds}$
4. [Kang et al., 2012] spatial modeling in spectral (time) domain allowing for correlated regions of interest (ROIs)
  - $N = 1$ , 4 sessions,  $V = 20/\text{ROI}$ , 3 ROIs,  $T = 480$

# SPM and FSL: Massive univariate analysis

- $n \in \{1, \dots, N\}$  denote subject
- $v \in \{1, \dots, V\}$  denote location (voxel for volume data, vertex for surface data)
- $t \in \{1, \dots, T\}$  index time
- $q \in \{1, \dots, Q\}$  index the task
- Task onsets and duration:  $I_{nq}(t)$ ; shape of the BOLD response to a stimulus:  $h(t)$ .

$$x_{ntq} = \int_0^{f_t} h(u) I_{nq}(f_t - u) du$$

where  $f_t$  is the time in seconds corresponding to the  $t$ th timepoint and we assume the same  $h(t)$  for all voxels.

# Hierarchical MUMM

$$(1 - \phi_{nv1}\mathbf{B} - \phi_{nv2}\mathbf{B}^2 - \phi_{nv3}\mathbf{B}^3) \{y_{nvt} - \mathbf{x}'_{nt}(\beta_v + \mathbf{b}_{nv}) - \mathbf{z}'_{nt}\gamma_{nv}\} \\ = \epsilon_{nvt}$$

with  $\mathbf{b}_{nv} \stackrel{iid}{\sim} \mathcal{N}\{\mathbf{0}, \text{diag}(\sigma_{b_1}^2, \dots, \sigma_{b_Q}^2)\}$  and  $\epsilon_{nvt} \stackrel{iid}{\sim} \mathcal{N}(0, \tau_{nv}^2)$ . All  $b_{nvq}$  and  $\epsilon_{nvt}$  are mutually independent.

Let  $\mathbf{B} = \text{diag}(\sigma_{b_1}^2, \dots, \sigma_{b_Q}^2)$ . Then,

$$\text{Cov } \mathbf{Y}_{nv} = \mathbf{X}_{nt}\mathbf{B}\mathbf{X}'_{nt} + \xi_{nv}^2\mathbf{\Psi}_{nv}. \quad (1)$$

## MUMM estimators

To simplify the exposition, assume  $\mathbf{z}_{nt} = 0$ . Define

$$\hat{\mathbf{e}}_{nv} = (\mathbf{X}'_n \mathbf{X}_n)^{-1} \mathbf{X}'_n \mathbf{Y}_{nv}. \quad (2)$$

Define the estimate of the population coefficients:

$$\hat{\beta}_v = \frac{1}{N} \sum_{n=1}^N \hat{\mathbf{e}}_{nv}. \quad (3)$$

Define the estimate of the variance of the fixed effect estimator:

$$\widehat{\text{Cov}} \hat{\beta}_v = \frac{1}{N(N-1)} \sum_{n=1}^N (\hat{\mathbf{e}}_{nv} - \hat{\beta}_v) (\hat{\mathbf{e}}_{nv} - \hat{\beta}_v)'. \quad (4)$$

# Spatiotemporal Mixed Model (STMM)

1. Automate smoothing using spatial random effects that capture population activation.

# Spatiotemporal Mixed Model (STMM)

1. Automate smoothing using spatial random effects that capture population activation.
2. Incorporate subject-vertex random effects to allow subject-specific deviations in activation and/or alignment.

# Spatiotemporal Mixed Model (STMM)

1. Automate smoothing using spatial random effects that capture population activation.
2. Incorporate subject-vertex random effects to allow subject-specific deviations in activation and/or alignment.
3. Unified model that includes subject- and vertex-specific autoregressive errors, which contrasts with previous methods that use the output from a first-level analysis.

# Spatiotemporal Mixed Model (STMM)

1. Automate smoothing using spatial random effects that capture population activation.
2. Incorporate subject-vertex random effects to allow subject-specific deviations in activation and/or alignment.
3. Unified model that includes subject- and vertex-specific autoregressive errors, which contrasts with previous methods that use the output from a first-level analysis.
4. Leverage improvements in registration using geodesic distances and improvements in parcellation using the Gordon parcels.

# Spatiotemporal Mixed Model (STMM)

1. Automate smoothing using spatial random effects that capture population activation.
2. Incorporate subject-vertex random effects to allow subject-specific deviations in activation and/or alignment.
3. Unified model that includes subject- and vertex-specific autoregressive errors, which contrasts with previous methods that use the output from a first-level analysis.
4. Leverage improvements in registration using geodesic distances and improvements in parcellation using the Gordon parcels.
5. Fast estimators of spatial dependence that can be used for whole-brain multi-subject studies.

# STMM

Define a separate model for each parcel.

To simplify the notation, let us consider a single region and ignore the index for region.

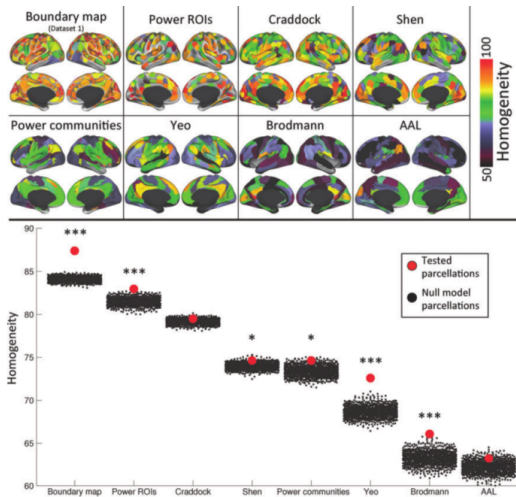
Let

$$\mathbf{Y}_n = [y_{n11}, \dots, y_{n1T}, y_{n21}, \dots, y_{n2T}, \dots, y_{nVT}]'$$

denote the BOLD signal for the  $n$ th subject at the vertices in our region, where  $V$  is the number of vertices in the region.

Let  $\mathbf{Y} = [\mathbf{Y}'_1, \dots, \mathbf{Y}'_N]'$ .

# [Gordon et al., 2014]



# STMM

$$y_{nvt} = \mathbf{x}'_{nt}(\boldsymbol{\beta} + \mathbf{u}_v + \mathbf{s}_n + \mathbf{b}_{nv}) + \mathbf{z}'_{nt}\boldsymbol{\gamma}_{nv} + a_{nvt}, \quad (5)$$

where

$$\mathbf{u}^q \sim \mathcal{N}(\mathbf{0}, \sigma_{u_q}^2 \boldsymbol{\Gamma}_q);$$

$$\mathbf{s}_n \stackrel{iid}{\sim} \mathcal{N}(\mathbf{0}, \mathbf{S})$$

with  $\mathbf{S} = \text{diag}(\sigma_{s_1}^2, \dots, \sigma_{s_Q}^2)$ ;

$$\mathbf{b}_n^q \stackrel{iid}{\sim} \mathcal{N}(\mathbf{0}, \sigma_{b_q}^2 \boldsymbol{\Omega}_q);$$

and

$$\mathbf{a} \sim \mathcal{N}(\mathbf{0}, \bigoplus_{n=1}^N \bigoplus_{v=1}^V \xi_{nv}^2 \boldsymbol{\Psi}_{nv}).$$

# Spatial Correlation

We assume the spatial correlation structure defined by exponential covariogram:

$$\Gamma_{q;vv'} = e^{-\theta_{bq}\|v-v'\|}$$

and

$$\Omega_{q;vv'} = e^{-\theta_{bq}\|v-v'\|}.$$

# Covariance Model

$\text{Cov } y_{nvt}, y_{n'v't'} =$

$$\begin{cases} \mathbf{x}'_{nt}(\mathbf{U} + \mathbf{S} + \mathbf{B})\mathbf{x}_{nt'} + \xi_{nv}^2 \psi_{tt'}^{(nv)} & n = n'; v = v'; \text{ any } t, t' \\ \mathbf{x}'_{nt}(\mathbf{U}\Gamma_{vv'} + \mathbf{S} + \mathbf{B}\Omega_{vv'})\mathbf{x}_{nt'} & n = n'; v \neq v'; \text{ any } t, t' \\ \mathbf{x}'_{nt}\mathbf{U}\Gamma_{vv'}\mathbf{x}_{n't'} & n \neq n'; \text{ any } v, v'; \text{ any } t, t' \end{cases}$$

No independence anywhere.

## ML and REML impossible

- We can not use maximum likelihood or restricted maximum likelihood methods to fit this model due to the enormous size of the covariance matrix, which is non-sparse for each parcel.

## ML and REML impossible

- We can not use maximum likelihood or restricted maximum likelihood methods to fit this model due to the enormous size of the covariance matrix, which is non-sparse for each parcel.
- In our application, the largest parcel's covariance matrix is 11 million by 11 million.

## ML and REML impossible

- We can not use maximum likelihood or restricted maximum likelihood methods to fit this model due to the enormous size of the covariance matrix, which is non-sparse for each parcel.
- In our application, the largest parcel's covariance matrix is 11 million by 11 million.
- The method we will propose involves a  $39,440 \times 39,440$  covariance matrix, which is about as much as we can handle with 32GB of RAM and computational shortcuts such as Sherman-Morrison-Woodbury.

## Second-level model

Transform the time dimension from  $T$  to the number of covariates  $Q$ :

$$\mathbf{d}_{nv} = \mathbf{K}_n' \mathbf{Y}_{nv}. \quad (6)$$

Define the second-level model

$$\begin{aligned} \mathbf{d} = & (\mathbf{1}_N \otimes \mathbf{1}_V \otimes \mathbf{I}_Q) \boldsymbol{\beta} + (\mathbf{1}_N \otimes \mathbf{I}_V \otimes \mathbf{I}_Q) \mathbf{u} + (\mathbf{I}_N \otimes \mathbf{1}_V \otimes \mathbf{I}_Q) \mathbf{s} \\ & + (\mathbf{I}_N \otimes \mathbf{I}_V \otimes \mathbf{I}_Q) \mathbf{b} + \{c \{c \mathbf{K}'_n \mathbf{a}_{nv}\}_{v=1}^V\}_{n=1}^N. \end{aligned}$$

## Estimation highlights

1. Bias-reduced estimators of AR parameters using OLS residuals and Yule-Walker equations (adapted from [Worsley et al., 2002])

## Estimation highlights

1. Bias-reduced estimators of AR parameters using OLS residuals and Yule-Walker equations (adapted from [Worsley et al., 2002])
2. Assume  $\Gamma_q = \Omega_q$ .

## Estimation highlights

1. Bias-reduced estimators of AR parameters using OLS residuals and Yule-Walker equations (adapted from [Worsley et al., 2002])
2. Assume  $\Gamma_q = \Omega_q$ .
3. Define a novel covariogram to allow vertex-specific temporal correlation

$$\delta_q(\mathbf{d}_{nvq}, \mathbf{d}_{nv'q}) = \begin{cases} \sigma_{u_q}^2 + \sigma_{s_q}^2 + \sigma_{b_q}^2 + \beta_q^2 & v = v', \\ (\sigma_{s_q}^2 + \beta_q^2) + (\sigma_{u_q}^2 + \sigma_{b_q}^2)e^{-\theta_q \|v - v'\|} & v \neq v'. \end{cases}$$

## Estimation highlights

1. Bias-reduced estimators of AR parameters using OLS residuals and Yule-Walker equations (adapted from [Worsley et al., 2002])
2. Assume  $\Gamma_q = \Omega_q$ .
3. Define a novel covariogram to allow vertex-specific temporal correlation

$$\delta_q(\mathbf{d}_{nvq}, \mathbf{d}_{nv'q}) = \begin{cases} \sigma_{u_q}^2 + \sigma_{s_q}^2 + \sigma_{b_q}^2 + \beta_q^2 & \mathbf{v} = \mathbf{v}', \\ (\sigma_{s_q}^2 + \beta_q^2) + (\sigma_{u_q}^2 + \sigma_{b_q}^2) e^{-\theta_q \|\mathbf{v} - \mathbf{v}'\|} & \mathbf{v} \neq \mathbf{v}'. \end{cases}$$

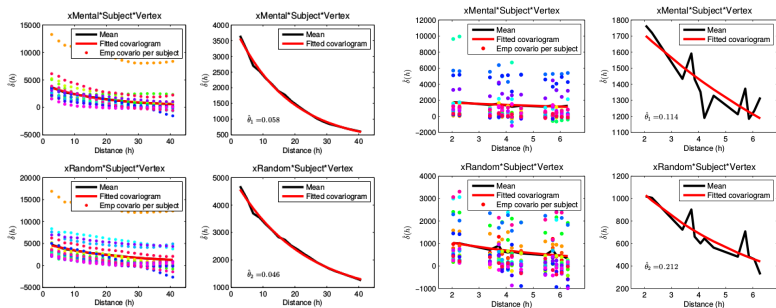
4. Define novel empirical covariogram for tolerance  $\nu$ :

$$\hat{\delta}_n(h) = \frac{1}{N_h} \sum_{\{\mathbf{v}, \mathbf{v}'\}: \|\mathbf{v} - \mathbf{v}'\| \in (h - \nu, h + \nu)} \mathbf{d}_{nvq} \mathbf{d}_{nv'q} \quad (7)$$

We can estimate  $\theta_q$  without knowing the other parameters!

# Estimating the spatial correlation parameter

Figure : Empirical covariogram and fitted exponential covariogram for the spatial random effects for a large (777 vertices) and very small (29 vertices) parcel.



# ANOVA estimators of variance components

Decompose the variance of the transformed data,  $\mathbf{d}_{nv} = \mathbf{K}_n' \mathbf{Y}_{nv}$ .

Source	DF	Sum of Squares
Vertex	V-1	$\sum_{n=1}^N \sum_{v=1}^V (\bar{\mathbf{d}}_{.v} - \bar{\mathbf{d}}_{..})(\bar{\mathbf{d}}_{.v} - \bar{\mathbf{d}}_{..})'$
Subject	N-1	$\sum_{n=1}^N \sum_{v=1}^V (\bar{\mathbf{d}}_{n.} - \bar{\mathbf{d}}_{..})(\bar{\mathbf{d}}_{n.} - \bar{\mathbf{d}}_{..})'$
Subject-Vertex	(N-1)(V-1)	$\sum_{n=1}^N \sum_{v=1}^V (\mathbf{d}_{nv} - \bar{\mathbf{d}}_{n.} - \bar{\mathbf{d}}_{.v} + \bar{\mathbf{d}}_{..})(\mathbf{d}_{nv} - \bar{\mathbf{d}}_{n.} - \bar{\mathbf{d}}_{.v} + \bar{\mathbf{d}}_{..})'$

# ANOVA estimators of variance components

Let  $g_q = \sum_{v=1}^V \sum_{v'=1}^V \Gamma_{q;v,v'}$  and  $\mathbf{G} = \text{diag}(g_1, \dots, g_Q)$ .

Let  $w_q = \sum_{v=1}^V \sum_{v'=1}^V \Omega_{q;v,v'}$  and  $\mathbf{W} = \text{diag}(w_1, \dots, w_Q)$ .

Source	EMS
Vertex	$\left(\frac{NV}{V-1}\mathbf{I}_Q - \frac{N}{V(V-1)}\mathbf{G}\right)\mathbf{U} + \left(\frac{V}{V-1}\mathbf{I}_Q - \frac{1}{V(V-1)}\mathbf{W}\right)\mathbf{B} + \frac{1}{NV} \sum_{n=1}^N \sum_{v'=1}^V \mathbf{K}_n \Psi_{nv'} \mathbf{K}'_n$
Subject	$V\mathbf{S} + \frac{1}{V}\mathbf{W}\mathbf{B} + \frac{1}{NV} \sum_{n=1}^N \sum_{v=1}^V \xi_{nv}^2 \mathbf{K}'_n \Psi_{nv} \mathbf{K}_n$
Subject-Vertex	$\left(\frac{V}{V-1}\mathbf{I}_Q - \frac{1}{V(V-1)}\mathbf{W}\right)\mathbf{B} + \frac{1}{NV} \sum_{n=1}^N \sum_{v=1}^V \xi_{nv}^2 \mathbf{K}'_n \Psi_{nv} \mathbf{K}_n$

## BLUEs, BLUPs, and eBLUPs

Define the BLUE for regional activation:

$$\hat{\beta} = \left\{ (\mathbf{1}'_N \otimes \mathbf{1}'_V \otimes \mathbf{I}_Q) \Sigma^{-1} (\mathbf{1}_N \otimes \mathbf{1}_V \otimes \mathbf{I}_Q) \right\}^{-1} (\mathbf{1}'_N \otimes \mathbf{1}'_V \otimes \mathbf{I}_Q) \Sigma^{-1} \mathbf{d} \quad (8)$$

$$\text{Cov } \hat{\beta} = \left\{ (\mathbf{1}'_N \otimes \mathbf{1}'_V \otimes \mathbf{I}_Q) \Sigma^{-1} (\mathbf{1}_N \otimes \mathbf{1}_V \otimes \mathbf{I}_Q) \right\}^{-1}.$$

Define the BLUP for vertex activation:

$$\hat{\mathbf{u}} = \Gamma (\mathbf{1}'_N \otimes \mathbf{I}_V \otimes \mathbf{I}_Q) \Sigma^{-1} \left( \mathbf{d} - \mathbf{1}_N \otimes \mathbf{1}_V \otimes \hat{\beta} \right). \quad (9)$$

We can also estimate  $\text{Cov } \hat{\mathbf{u}}$ .

## Approximate inference

$$[\mathbf{u}|\mathbf{d}, \widehat{\Sigma}, \widehat{\beta}] \sim \mathcal{N} \left( \widehat{\Gamma} \{ \mathbf{1}'_N \otimes \mathbf{I}_V \otimes \mathbf{I}_Q \} \widehat{\Sigma}^{-1} \left( \mathbf{d} - \mathbf{1}_N \otimes \mathbf{1}_V \otimes \widehat{\beta} \right), \right. \\ \left. \widehat{\Gamma} - \widehat{\Gamma} \{ \mathbf{1}'_N \otimes \mathbf{I}_V \otimes \mathbf{I}_Q \} \widehat{\Sigma}^{-1} \{ \mathbf{1}_N \otimes \mathbf{I}_V \otimes \mathbf{I}_Q \} \widehat{\Gamma} \right).$$

For an arbitrary contrast vector  $\mathbf{c}$ ,

$$H_0 : \mathbf{c}'[\beta', E(\mathbf{u}|\mathbf{d})']' = 0$$

$$H_A : \mathbf{c}'[\beta', E(\mathbf{u}|\mathbf{d})']' \neq 0.$$

We use  $\widehat{\mathbf{u}}$  as our estimate of  $E(\mathbf{u}|\mathbf{d})$ . Approximate t-statistic:

$$t = \frac{\mathbf{c}'[\widehat{\beta}', \widehat{\mathbf{u}}']'}{\left\{ \mathbf{c}'(\widehat{\text{Cov}} \widehat{\beta} \oplus \widehat{\text{Cov}} \widehat{\mathbf{u}})\mathbf{c} \right\}^{1/2}}.$$

## Estimators are approximately unbiased

	Truth	df	Mean STMM	Var STMM	Bias <sup>2</sup> STMM	MSE STMM
$\tau_{nv}^2$	30000	-	29811	3434698	35882	3470580
$\phi_{nv1}$	0.2000	-	0.1988	0.0020	0.0000	0.0020
$\phi_{nv2}$	0.1000	-	0.0969	0.0020	0.0000	0.0021
$\phi_{nv3}$	0.0000	-	-0.0008	0.0020	0.0000	0.0020
$\beta_1$	20.0	-	19.9	16.2	0.0	16.2
$\sigma_{u_1}^2$	75.0	214	77.0	776.0	4.1	780.2
$\sigma_{s_1}^2$	250	29	241	8521	84	8605
$\sigma_{b_1}^2$	750	6206	750	3517	0	3517
$\theta_{b_1}$	0.200	-	0.201	0.002	0.000	0.002
$\beta_2$	-20.0	-	-20.0	17.5	0.0	17.5
$\sigma_{u_2}^2$	75.0	214	75.9	808.2	0.9	809.0
$\sigma_{s_2}^2$	250	29	244	7947	38	7985
$\sigma_{b_2}^2$	750	6206	749	3747	2	3749
$\theta_{b_2}$	0.200	-	0.203	0.002	0.000	0.002

**Table :** Accuracy of estimators for 1000 simulations with 30 subjects each in which 215 vertices were located according to a Gordon Parcel (ID 82).

# STMM: more power with no $\mathbf{U}$ ; less power with $\mathbf{U}$

Scenario	Model	Region			Vertex		
		$\beta_1 = 20$ (Power)	$\beta_2 = 0$ (Type 1)	Contrast (Power)	$\beta_1 + u_{v1}$ (Power)	$\beta_2 + u_{v2}$ (Type 1*)	Contrast (Power)
$\theta_1 = \theta_2 = 5$ $\sigma_{u_1}^2 = \sigma_{u_2}^2 = 0$	MUMM	1.00	0.72	1.00	0.61	0.05	0.38
	ROIMM	1.00	0.05	0.99	-	-	-
	STMM	1.00	0.06	0.99	1.00	0.02	0.99
$\theta_1 = \theta_2 = 5$ $\sigma_{u_1}^2 = \sigma_{u_2}^2 = 100$	MUMM	1.00	0.70	1.00	0.57	0.19	0.41
	ROIMM	1.00	0.05	1.00	-	-	-
	STMM	1.00	0.06	1.00	0.63	0.02	0.35
$\theta_1 = \theta_2 = 0.2$ $\sigma_{u_1}^2 = \sigma_{u_2}^2 = 0$	MUMM	1.00	0.72	1.00	0.61	0.05	0.39
	ROIMM	1.00	0.05	0.99	-	-	-
	STMM	1.00	0.04	0.99	1.00	0.02	0.98
$\theta_1 = \theta_2 = 0.2$ $\sigma_{u_1}^2 = \sigma_{u_2}^2 = 100$	MUMM	1.00	0.79	1.00	0.56	0.19	0.42
	ROIMM	1.00	0.14	0.95	-	-	-
	STMM	0.99	0.03	0.86	0.57	0.02	0.31

Table : Power and type 1 error rates based on 300 simulations for each scenario.

## (xMental - xRandom) from HCP ToM

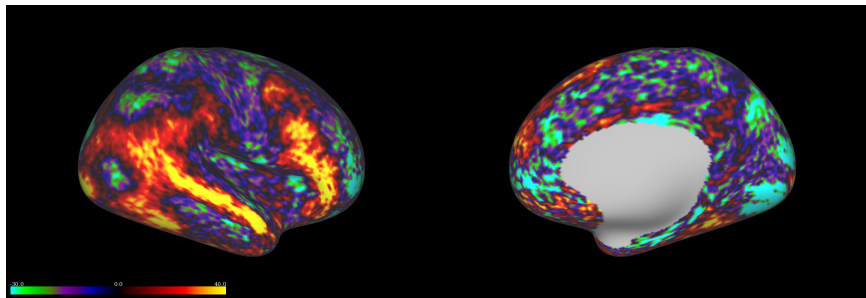


Figure : Contrast in activation from the mentalizing versus random tasks from the MUMM.

## (xMental - xRandom) from HCP ToM

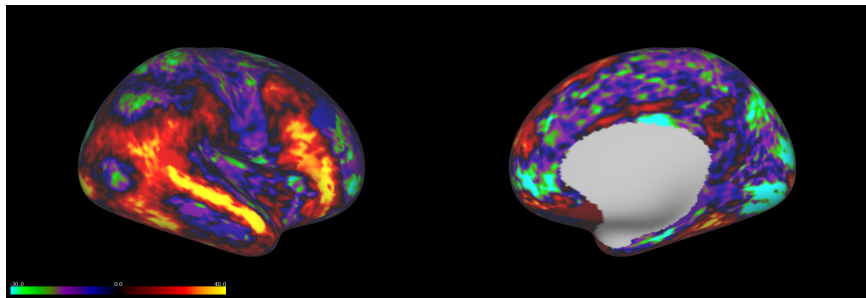


Figure : Contrast in activation from the mentalizing versus random tasks from the STMM.

# t-statistic (xMental-xRandom) from HCP ToM

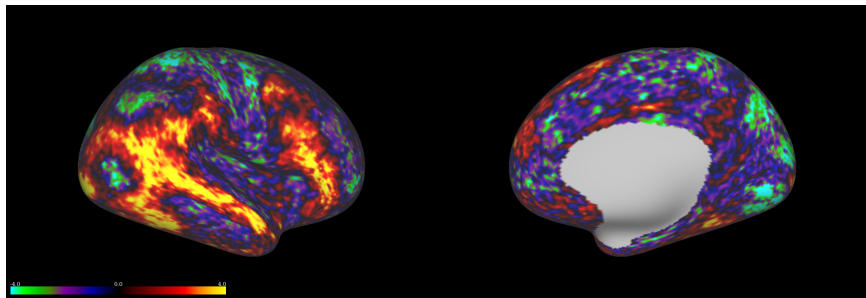


Figure : The contrast t-statistic from the MUMM

# t-statistic (xMental-xRandom) from HCP ToM

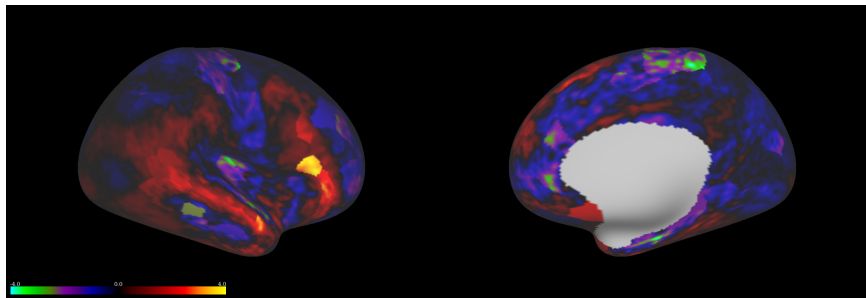


Figure : The contrast t-statistic from the STMM

# Summary of Spatiotemporal Mixed Model (STMM)

1. Automate smoothing.

# Summary of Spatiotemporal Mixed Model (STMM)

1. Automate smoothing.
2. Subject-vertex random effects allow subject-specific deviations in activation and/or alignment.

# Summary of Spatiotemporal Mixed Model (STMM)

1. Automate smoothing.
2. Subject-vertex random effects allow subject-specific deviations in activation and/or alignment.
3. Unified model that includes subject- and vertex-specific autoregressive errors, which contrasts with previous methods that use the output from a first-level analysis.

# Summary of Spatiotemporal Mixed Model (STMM)

1. Automate smoothing.
2. Subject-vertex random effects allow subject-specific deviations in activation and/or alignment.
3. Unified model that includes subject- and vertex-specific autoregressive errors, which contrasts with previous methods that use the output from a first-level analysis.
4. Leverage improvements in cortical registration and improvements in parcellation by using the geodesic distances between vertices within a Gordon parcel.

# Summary of Spatiotemporal Mixed Model (STMM)

1. Automate smoothing.
2. Subject-vertex random effects allow subject-specific deviations in activation and/or alignment.
3. Unified model that includes subject- and vertex-specific autoregressive errors, which contrasts with previous methods that use the output from a first-level analysis.
4. Leverage improvements in cortical registration and improvements in parcellation by using the geodesic distances between vertices within a Gordon parcel.
5. Fast estimators of spatial dependence.

## Discussion and the future

1. Preliminary research: a simpler model model with vertex fixed effects is more powerful method than MUMM or STMM, but don't get population effects.

## Discussion and the future

1. Preliminary research: a simpler model model with vertex fixed effects is more powerful method than MUMM or STMM, but don't get population effects.
2. STMM could be used to improve pre-surgical mapping in an individual by shrinking towards population effects.

## Discussion and the future

1. Preliminary research: a simpler model model with vertex fixed effects is more powerful method than MUMM or STMM, but don't get population effects.
2. STMM could be used to improve pre-surgical mapping in an individual by shrinking towards population effects.
3. Inference for any spatial domain: space is continuous in STMM, easy extension.

## Discussion and the future

1. Preliminary research: a simpler model model with vertex fixed effects is more powerful method than MUMM or STMM, but don't get population effects.
2. STMM could be used to improve pre-surgical mapping in an individual by shrinking towards population effects.
3. Inference for any spatial domain: space is continuous in STMM, easy extension.
4. Examine residuals for mid/long range dependencies and model these correlations.

## Discussion and the future

1. Preliminary research: a simpler model model with vertex fixed effects is more powerful method than MUMM or STMM, but don't get population effects.
2. STMM could be used to improve pre-surgical mapping in an individual by shrinking towards population effects.
3. Inference for any spatial domain: space is continuous in STMM, easy extension.
4. Examine residuals for mid/long range dependencies and model these correlations.
5. STMM could be used to examine how spatial correlation varies by ROI.

## Acknowledgements

Thank you to my advisors, David Matteson and David Ruppert. Thank you to Nathan Spreng for scientific input. Data were provided (in part) by the Human Connectome Project, WU-Minn Consortium (Principal Investigators: David Van Essen and Kamil Ugurbil; 1U54MH091657) funded by the 16 NIH Institutes and Centers that support the NIH Blueprint for Neuroscience Research; and by the McDonnell Center for Systems Neuroscience at Washington University.

# References I



Bernal-Rusiel, J. L., Reuter, M., Greve, D. N., Fischl, B., and Sabuncu, M. R. (2013).  
Spatiotemporal linear mixed effects modeling for the mass-univariate analysis of longitudinal neuroimage data.  
*NeuroImage*, 81:358–370.



Bowman, F. D. (2007).  
Spatiotemporal models for region of interest analyses of functional neuroimaging data.  
*Journal of the American Statistical Association*, 102(478):442–453.



Derado, G., Bowman, F. D., and Kilts, C. D. (2010).  
Modeling the spatial and temporal dependence in fMRI data.  
*Biometrics*, 66(3):949–957.



Gordon, E. M., Laumann, T. O., Adeyemo, B., Huckins, J. F., Kelley, W. M., and Petersen, S. E. (2014).  
Generation and evaluation of a cortical area parcellation from resting-state correlations.  
*Cerebral Cortex*, page bhu239.



Kang, H., Ombao, H., Linkletter, C., Long, N., and Badre, D. (2012).  
Spatio-spectral mixed-effects model for functional magnetic resonance imaging data.  
*Journal of the American Statistical Association*, 107(498):568–577.



Worsley, K. J., Liao, C., Aston, J., Petre, V., Duncan, G., Morales, F., and Evans, A. (2002).  
A general statistical analysis for fmri data.  
*Neuroimage*, 15(1):1–15.